Indian Statistical Institute, Bangalore Centre
M.Math. (I Year) : 2015-2016
Semester I : Mid-Semestral Examination
Measure theoretic Probability

11.09.2015 Time: $2\frac{1}{2}$ hours. Maximum Marks : 80

Note: Notation and terminology are understood to be as used in class. State clearly the results you are using in your answers.

1. (15 marks) Let $(\Omega, \mathcal{F}, \mu)$ be a Boolean measure space; let μ^* denote the corresponding outer measure. If $\mu^*(E) = 0$, show that E is μ^*-measurable.

2. (15 marks) Let f be a real valued function on a measurable space $(\Omega, \mathcal{B})$. Show that f is measurable if and only if $E_r \in \mathcal{B}$ for any rational number r, where $E_r = \{\omega \in \Omega : f(\omega) \leq r\}$.

3. (15 marks) Let $(\Omega, \mathcal{B}, \mu)$ be a $\sigma-$finite measure space, and let f be a nonnegative integrable function on it. Define $\lambda(E) = \int_E f(\omega) d\mu(\omega)$ for $E \in \mathcal{B}$. Show that λ is a totally finite measure on $(\Omega, \mathcal{B})$.

4. ($10 + 10 + 15 = 35$ marks) Let f be a real valued measurable function on a $\sigma-$finite measure space $(\Omega, \mathcal{B}, \mu)$. For $n = 1, 2, \cdots$ define

$$f_n(\omega) = \begin{cases} -n, \text{ if } f(\omega) < (-n), \\ f(\omega), \text{ if } |f(\omega)| \leq n, \\ n, \text{ if } f(\omega) > n. \end{cases}$$

(i) Show that f_n is a measurable function for each n.

(ii) If f is integrable with respect to μ, show that $\int_\Omega f(\omega) d\mu(\omega) = \lim_{n\to\infty} \int_\Omega f_n(\omega) d\mu(\omega)$.

(iii) If $\sup_n \int_\Omega |f_n(\omega)| d\mu(\omega) < \infty$, show that f is integrable with respect to μ.